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Three-dimensional Poincaré supergravity and $\mathcal{N}$-extended supersymmetric BMS$_3$ algebra

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ABSTRACT

A new approach for obtaining the three-dimensional Chern-Simons supergravity for the Poincaré algebra is presented. The $\mathcal{N}$-extended Poincaré supergravity is obtained by expanding the super Lorentz theory. We extend our procedure to their respective asymptotic symmetries and show that the $\mathcal{N} = (1, 2, 4)$ super-BMS$_3$ appear as expansions of one Virasoro superalgebra. Interestingly, the $\mathcal{N}$-extended super-BMS$_3$ obtained here are not only centrally extended but also endowed with internal symmetry. We also show that the $\mathcal{N}$-extended super Poincaré algebras with both central and automorphism generators are finite subalgebras.

1. Introduction

Three-dimensional (super)gravity theories result of particular interest since they represent attractive toy models for understanding richer (super)gravities. Indeed, there are still open issues to solve in higher dimensions that motivate to study the three-dimensional case. In the last decades, diverse supergravity models have been presented in three spacetime dimensions in [1–16]. In particular, $\mathcal{N}$-extended three-dimensional supergravity without cosmological constant [3] can be expressed as a Chern-Simons (CS) action for the Poincaré supergroup [17]. Subsequently, a new class of (p, q)-extended Poincaré CS supergravities has been constructed in [18]. Interestingly, such $\mathcal{N}$-extended flat supergravity with both central and automorphism charges emerges properly as a vanishing cosmological constant limit of an $\mathcal{N}$-extended AdS$_3$ supergravity [18].

Recently, there has been a particular interest in the infinite-dimensional symmetries of asymptotically flat spacetimes at null infinity which were proposed to be spanned by the BMS algebra originally discovered more than a half century ago [19,20]. In three dimensions, it has been shown in [21–23] that the asymptotically flat symmetry is described by the BMS$_3$ algebra. Such infinite-dimensional algebra results to be isomorphic to the Galilean conformal algebra in two dimensions [24]. Interestingly, the BMS$_3$ algebra appears as a flat limit of the conformal algebra which describe the asymptotic symmetries of three-dimensional gravity [25]. More recently, it was shown in [26] that the BMS$_3$ algebra can be alternatively derived as an algebraic expansion of the Virasoro one. The derivation of the BMS$_3$ symmetry using an algebraic operation has also been considered in [27]. Further extensions and deformations of the BMS$_3$ algebra have been recently studied in [28–38].

At the supersymmetric level, a minimal supersymmetric extension of BMS$_3$ appears as the asymptotic symmetry of a three-dimensional $\mathcal{N} = 1$ supergravity for suitable boundary conditions [39]. Such superalgebra turns out to be isomorphic to the Galilean superconformal algebra [40,41]. The supersymmetric extension to $\mathcal{N} = 2$ [42,43], $\mathcal{N} = 4$ [44] and $\mathcal{N} = 8$ [45] of the BMS$_3$ has been subsequently explored by an asymptotic symmetry analysis. Remarkably, the $\mathcal{N}$-extended super-BMS$_3$ can be obtained by an appropriate contraction of the $\mathcal{N}$-extended superconformal algebras [46].

In this paper, we present a novel approach to obtain the $\mathcal{N}$-extended super-BMS$_3$ algebra by considering the semigroup expansion method (S-expansion) [47]. The algebraic expansion methods [47–49] have been particularly useful in the context of (super)gravity theories [50–68]. In three dimensions, the S-expansion procedure has not only allowed to reproduce known (super)gravity theories [69] but also to obtain novel (super)gravity actions [70–72]. Here, we will show first that the three-dimensional $\mathcal{N}$-extended Poincaré CS supergravity theory can be alternatively

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derived from a super-Lorentz CS theory using a particular semigroup. Interestingly, such procedure can be extended to infinite-dimensional algebras allowing us to reproduce the $\mathcal{N}$-extended super-BMS_3 algebra from the super Virasoro algebra using the same finite semigroup. Let us note that the procedure considered here, unlike the contraction, requires only one $\mathcal{N}$-extended super Virasoro algebra instead of two copies. In particular, the $\mathcal{N}>1$ super-BMS_3 algebras obtained here are non only centrally extended but also contain internal symmetry algebra. Thus, the finite subalgebra corresponds to the $\mathcal{N}$-extended Poincaré superalgebra endowed with automorphism generators. It is important to point out that the $\mathcal{N}=2$ super-BMS_3 presented here can be easily recovered from the $\mathcal{N}=4$ one presented in [46] after setting some fermionic generators to zero. The results obtained here can be seen as a supersymmetric generalization of those presented in [26] and could be useful to derive new infinite-dimensional superalgebras.

The paper is organized as follows: In Section 2, we apply the S-expansion method to obtain the $\mathcal{N}$-extended Poincaré supergravity from an $\mathcal{N}$-extended super-Lorentz theory in three spacetime dimensions. A brief introduction to the $\mathcal{N}$-extended super-Lorentz CS theory is also presented. In Section 3, we extend our procedure to infinite-dimensional superalgebras. In particular, the $\mathcal{N}$-extended super-BMS_3 are obtained from an $\mathcal{N}$-extended super-Virasoro algebra for $\mathcal{N}=1,2$ and 4. We show that the $\mathcal{N}$-extended super-Poincaré algebra is a finite subalgebra of the supersymmetric extension of the BMS_3 algebra. We also discuss the presence of internal symmetry algebra in the $\mathcal{N}$-extended super-BMS_3 obtained here for $\mathcal{N}>1$. In Section 4, we conclude with some comments about possible developments and extensions of our results.

2. $\mathcal{N}$-extended Poincaré supergravity and super-Lorentz theory in three spacetime dimensions

The possibility of having a well defined three-dimensional AdS CS gravity action from a Lorentz action using the S-expansion procedure has been considered in [53]. Here, we extend such result to the $\mathcal{N}$-extended Poincaré CS supergravity in three spacetime dimensions. In particular, the three-dimensional $\mathcal{N}$-extended Poincaré superalgebra can be obtained from a supersymmetric extension of the Lorentz algebra using the S-expansion. Such method allows us to obtain the non-vanishing components of the invariant tensor of the super Poincaré which are essential to construct a CS action. However, it is important to point out that the procedure presented here can be applied only in three spacetime dimensions. This particular accident comes from the fact that the expanded Lorentz generators can be interpreted as translational generators.

2.1. $\mathcal{N}$-extended super-Lorentz theory

A supersymmetric extension of the Lorentz algebra in three spacetime dimensions is generated by the bosonic set $\left\{M_a,\ \tilde{T}^{ij}\right\}$ and Majorana fermionic generators $\left\{\tilde{Q}^{i}\right\}$ with $i,j=1,\dots,\mathcal{N}$. The non-vanishing (anti-)commutation relations of an $\mathcal{N}$-extended super-Lorentz algebra read

$$\begin{aligned}
&[M_a, M_b] = \epsilon_{abc}M^c\,,\\
&\left[M_a, \tilde{Q}^i_\alpha\right] = \frac{1}{2}\left(\Gamma_a\right)^\beta_\alpha \tilde{Q}^i_\beta\,,\quad \left[\tilde{T}^{ij}, \tilde{Q}^k_\alpha\right] = \left(\delta^{jk}\tilde{Q}^i_\alpha - \delta^{ik}\tilde{Q}^j_\alpha\right),\\
&\left[\tilde{T}^{ij}, \tilde{T}^{kl}\right] = \delta^{jk}\tilde{T}^{il} - \delta^{ik}\tilde{T}^{jl} - \delta^{jl}\tilde{T}^{ik} + \delta^{il}\tilde{T}^{jk}\,,\\
&\left\{\tilde{Q}^i_\alpha, \tilde{Q}^j_\beta\right\} = -\frac{1}{2}\delta^{ij}\left(C\Gamma^a\right)_{\alpha\beta}M_a + C_{\alpha\beta}\tilde{T}^{ij}\,,
\end{aligned} \tag{2.1}$$

where $a,b,\dots=0,1,2$ are Lorentz indices, Γ_a denote the Dirac matrices and C represents the charge conjugation matrix,

$$C_{\alpha\beta} = C^{\alpha\beta} = \begin{pmatrix} 0 & -1\\ 1 & 0\end{pmatrix}. \tag{2.2}$$

In particular, the Dirac Matrices can be written in terms of the Pauli matrices σ_i as

$$\Gamma_0 = \tfrac{1}{\sqrt{2}}\left(\sigma_1 + i\sigma_2\right)\,,\ \Gamma_1 = \tfrac{1}{\sqrt{2}}\left(\sigma_1 - i\sigma_2\right)\,,\ \Gamma_2 = \sigma_3\,, \tag{2.3}$$

with

$$\sigma_1 = \begin{pmatrix} 0 & 1\\ 1 & 0\end{pmatrix}\,,\ \sigma_2 = \begin{pmatrix} 0 & -i\\ i & 0\end{pmatrix}\,,\ \sigma_3 = \begin{pmatrix} 1 & 0\\ 0 & -1\end{pmatrix}. \tag{2.4}$$

Here $\tilde{T}^{ij} = -\tilde{T}^{ji}$ are internal symmetry generators with $i=1,\dots,\mathcal{N}$.

Let us note that the Lorentz superalgebra introduced in [73] corresponds to the $\mathcal{N}=1$ case and it is spanned by the set of generators $\left\{M_a,\ \tilde{Q}_\alpha\right\}$. On the other hand, the $\mathcal{N}=2$ Lorentz superalgebra implies the introduction of the $\mathfrak{so}(2)$ automorphism algebra through the $\tilde{T}$ generator.

A CS action can be constructed from the Lorentz supergroup which has the following non-vanishing components of the invariant tensor,

$$\begin{aligned}
&\langle M_a M_b\rangle = \eta_{ab}\,,\\
&\left\langle \tilde{Q}^i_\alpha \tilde{Q}^j_\beta\right\rangle = C_{\alpha\beta}\delta^{ij}\,,\\
&\left\langle \tilde{T}^{ij}\tilde{T}^{kl}\right\rangle = \delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\,,
\end{aligned} \tag{2.5}$$

where η_{ab} is the Minkowski metric. On the other hand, let us consider the gauge connection one-form A,

$$A = \omega^a M_a + \bar{\psi}^i\tilde{Q}^i + \frac{1}{2}A^{ij}\tilde{T}_{ij}\,, \tag{2.6}$$

where the coefficients in front of the generators correspond to the gauge potential one-forms. In particular, the gauge fields ω^a, ψ are the spin connection and the Majorana spinor field, respectively. The field strength two-form $F = dA + \frac{1}{2}[A, A]$ reads

$$F = F^a M_a + \nabla\bar{\psi}^i\tilde{Q}^i + F^{ij}\tilde{T}^{ij}\,, \tag{2.7}$$

with

$$\begin{aligned}
&F^a = d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b\omega_c + \frac{1}{4}\bar{\psi}^i\Gamma^a\psi^i\,,\\
&F^{ij} = dA^{ij} + A^{ik}A^{kj} + \bar{\psi}^i\psi^j\,.
\end{aligned} \tag{2.8}$$

Here, the covariant derivative acting on spinors reads

$$\nabla\psi^i = d\psi^i + \frac{1}{2}\omega^a\Gamma_a\psi^i + A^{ij}\psi^j\,. \tag{2.9}$$

The CS action

$$I_{CS} = \frac{k}{4\pi}\int\left\langle AdA + \frac{2}{3}A^3\right\rangle, \tag{2.10}$$

can be written, using the invariant tensor (2.5) and the connection one-form (2.6) as

$$I^{(2+1)}_{CS} = \frac{k}{4\pi}\int \omega^a d\omega_a + \frac{1}{3}\epsilon_{abc}\omega^a\omega^b\omega^c - \frac{1}{2}\mathcal{G}(A^{ij}) - \bar{\psi}^i\nabla\psi^i\,, \tag{2.11}$$

where the three-form $\mathcal{G}$ is defined as

$$\mathcal{G}\left(A^{ij}\right)=A^{ij}dA^{ji}+\frac{2}{3}A^{ik}A^{km}A^{mi}\,. \tag{2.12}$$

The field equations are given by the vanishing of the coefficients appearing in the field strength (2.7)

$$F^{a}=0\,,\qquad \nabla\psi^{i}=0\,,\qquad F^{ij}=0\,. \tag{2.13}$$

Note that the Lagrangian (2.11) contains the exotic Lagrangian, also known as Lorentz Lagrangian, $L_{exotic}=\omega^{a}d\omega_{a}+\frac{1}{3}\epsilon_{abc}\omega^{a}\omega^{b}\omega^{c}$. The above three-dimensional action describes the coupling of Majorana spinor field to the exotic term and to an $SO\left(\mathcal{N}\right)$ CS term. Such action is invariant, up to boundary terms, under the gauge transformation $\delta A=D\lambda=d\lambda+[A,\lambda]$. In particular, the non-zero supersymmetry transformation laws are given by

$$\begin{aligned}
\delta\omega^{a}&=\frac{1}{2}\bar{\epsilon}^{i}\Gamma^{a}\psi^{i}\,,\\
\delta\psi^{i}&=\nabla\epsilon^{i}\,,\\
\delta A^{ij}&=-2\bar{\psi}^{[i}\epsilon^{j]}\,,
\end{aligned} \tag{2.14}$$

where one can see that the $SO\left(\mathcal{N}\right)$ automorphism gauge fields are present. It is no a surprise that the action (2.11) is invariant under the super-Lorentz group since it is built from the gauge connection one-form A as a CS action.

2.2. $\mathcal{N}$-extended Poincaré supergravity theory from super-Lorentz

The $\mathcal{N}$-extended CS Poincaré supergravity theory in three spacetime dimensions in presence of both central and automorphism charges, has been carefully studied in [18]. Although the $\mathcal{N}$-extended Poincaré supergravity has been discussed much earlier by diverse authors [3,17], the $\mathcal{N}=p+q$ super Poincaré one presented in [18] has interesting advantages. In particular, the (p,q)-extended Poincaré supergravity appears as a Poincaré limit of the corresponding (p,q)-extended AdS supergravity theories [18,69]. Additionally, the $(1,1)$ and $(2,0)$ super Poincaré theories possess an off-shell superfield formulation.

In this section, following a similar procedure used in [26], we present a novel method to recover the $\mathcal{N}$-extended Poincaré supergravity theory. We show that an algebraic expansion mechanism can be applied to the super-Lorentz theory introduced in the previous section. Such method allows not only to recover the complete set of (anti-)commutation relations of the $\mathcal{N}$-extended Poincaré superalgebra (including both central and automorphism charges) but also the complete CS supergravity action. As we shall see in the next section, the procedure used here can be generalized at the asymptotic level.

The super-Lorentz algebra can be decomposed in subspaces as

$$s\mathcal{L}=V_{0}\oplus V_{1}\,, \tag{2.15}$$

where V_0 is the bosonic subspace spanned by the Lorentz generator M_a and the automorphism generators $\tilde{T}^{ij}$. On the other hand, V_1 is the fermionic subspace. Such subspaces satisfy a graded Lie algebra,

$$\begin{aligned}
\left[V_{0},V_{0}\right]&\subset V_{0}\,,\\
\left[V_{0},V_{1}\right]&\subset V_{1}\,,\\
\left[V_{1},V_{1}\right]&\subset V_{0}\,.
\end{aligned} \tag{2.16}$$

Interestingly, the $\mathcal{N}$-extended super Poincaré structure can be recovered from it using the S-expansion method [47]. However, it is necessary that the pertinent semigroup S possess a particular decomposition $S=S_0\cup S_1$ which has to behave as the subspaces of the super-Lorentz algebra. An abelian semigroup with the desired behavior is $S_E^{(2)}=\{\lambda_0,\lambda_1,\lambda_2,\lambda_3\}$ whose elements satisfy

$$\begin{array}{c|cccc}
\lambda_3 & \lambda_3 & \lambda_3 & \lambda_3 & \lambda_3\\
\lambda_2 & \lambda_2 & \lambda_3 & \lambda_3 & \lambda_3\\
\lambda_1 & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_3\\
\lambda_0 & \lambda_0 & \lambda_1 & \lambda_2 & \lambda_3\\
\hline
 & \lambda_0 & \lambda_1 & \lambda_2 & \lambda_3
\end{array} \tag{2.17}$$

where $\lambda_3=0_s$ is the zero element of the semigroup. In particular, the following subset decomposition $S_E^{(2)}=S_0\cup S_1$, with

$$\begin{aligned}
S_0&=\{\lambda_0,\lambda_2,\lambda_3\}\,,\\
S_1&=\{\lambda_1,\lambda_3\}\,,
\end{aligned} \tag{2.18}$$

is said to be resonant since it satisfies the same subspace's structure,

$$\begin{aligned}
S_0\cdot S_0&\subset S_0\,,\\
S_0\cdot S_1&\subset S_1\,,\\
S_1\cdot S_1&\subset S_0\,.
\end{aligned} \tag{2.19}$$

Following the definitions of [47], after extracting a resonant subalgebra of $S_E^{(2)}\times s\mathcal{L}$ and applying its 0_s-reduction, one finds an expanded superalgebra whose generators $\left\{J_a,P_a,T^{ij},Z^{ij},Q^{i}\right\}$ are related to the super-Lorentz one as

$$\begin{aligned}
J_a&=\lambda_0 M_a\,,\\
P_a&=\lambda_2 M_a\,,\\
T^{ij}&=\lambda_0\tilde{T}^{ij}\,,\\
Z^{ij}&=\lambda_2\tilde{T}^{ij}\,,\\
Q^{i}&=\lambda_1\tilde{Q}^{i}\,.
\end{aligned} \tag{2.20}$$

Using the multiplication law of the semigroup, one can see that the generators of the expanded superalgebra satisfy the following non-vanishing (anti-)commutation relations

$$\begin{aligned}
\left[J_a,J_b\right]&=\epsilon_{abc}J^{c}\,,\\
\left[J_a,P_b\right]&=\epsilon_{abc}P^{c}\,,\\
\left[J_a,Q_{\alpha}^{i}\right]&=\frac{1}{2}\left(\Gamma_a\right)_{\alpha}^{\beta}Q_{\beta}^{i}\,,\\
\left\{Q_{\alpha}^{i},Q_{\beta}^{j}\right\}&=-\frac{1}{2}\delta^{ij}\left(C\Gamma^{a}\right)_{\alpha\beta}P_a+C_{\alpha\beta}Z^{ij}\,,
\end{aligned} \tag{2.21}$$

$$\begin{aligned}
\left[T^{ij},Q_{\alpha}^{k}\right]&=\left(\delta^{jk}Q_{\alpha}^{i}-\delta^{ik}Q_{\alpha}^{j}\right)\,,\\
\left[T^{ij},T^{kl}\right]&=\delta^{jk}T^{il}-\delta^{ik}T^{jl}-\delta^{jl}T^{ik}+\delta^{il}T^{jk}\,,\\
\left[T^{ij},Z^{kl}\right]&=\delta^{jk}Z^{il}-\delta^{ik}Z^{jl}-\delta^{jl}Z^{ik}+\delta^{il}Z^{jk}\,.
\end{aligned} \tag{2.22}$$

The (anti-)commutation relations given by (2.21) correspond to the central extension of the $\mathcal{N}$-extended Poincaré superalgebra. Interestingly, the S-expansion procedure also provides with the automorphism charges which satisfy (2.22). In particular, unlike the contraction method in which some (anti-)commutators vanish, the expansion provides us with a bigger algebra with a new set of (anti-)commutators. Here, the set of (anti-)commutation relations is known as the complete $\mathcal{N}$-extended Poincaré superalgebra [18].

As was mentioned in [18], the superalgebra (2.21) does not admit a non-degenerate invariant inner product which prevent the formulation of a CS supergravity action. It is the presence of the automorphism group which allows a CS supergravity formulation.

Note that the central charges Z^{ij} do not need to be invariant under the automorphism algebra.

Remarkably, the S-expansion does not limit only to the obtention of the super-Poincaré generators but also provides us with the non-vanishing components of the invariant tensor of the $\mathcal{N}$-extended Poincaré supergravity action. In fact, following the definitions of [47], the Poincaré invariant tensor is given in term of the super-Lorentz one:

$$\begin{aligned}
\langle J_a J_b\rangle &= \mu_0 \langle M_a M_b\rangle = \mu_0\eta_{ab}\,,\\
\langle J_a P_b\rangle &= \mu_2 \langle M_a M_b\rangle = \mu_2\eta_{ab}\,,\\
\left\langle Q^i_\alpha Q^j_\beta\right\rangle &= \mu_2\left\langle \tilde{Q}^i_\alpha \tilde{Q}^j_\beta\right\rangle = \mu_2 C_{\alpha\beta}\delta^{ij}\,,\\
\left\langle T^{ij}T^{kl}\right\rangle &= \mu_0\left\langle \tilde{T}^{ij}\tilde{T}^{kl}\right\rangle = \mu_0\left(\delta^{ik}\delta^{lj}-\delta^{il}\delta^{kj}\right),\\
\left\langle Z^{ij}T^{kl}\right\rangle &= \mu_2\left\langle \tilde{T}^{ij}\tilde{T}^{kl}\right\rangle = \mu_2\left(\delta^{ik}\delta^{lj}-\delta^{il}\delta^{kj}\right),
\end{aligned} \tag{2.23}$$

where μ_0 and μ_2 are arbitrary constants.

The connection one-form reads

$$A=\omega^a J_a + e^a P_a + \bar{\psi}^i Q^i + \frac{1}{2}A^{ij}T_{ij} + \frac{1}{2}C^{ij}Z_{ij}\,, \tag{2.24}$$

where the coefficients in front of the generators are the gauge potential one-forms. Let us note that the vielbein e^a, which was not present in the super Lorentz theory, appears naturally from the S-expansion procedure. Furthermore, the methodology provides us with central gauge fields C^{ij} in addition to the automorphism gauge fields A^{ij}.

The curvature two-form is given by

$$F=R^a J_a + T^a P_a + \nabla\bar{\psi}^i Q^i + \frac{1}{2}F^{ij}T_{ij} + \frac{1}{2}G^{ij}Z_{ij}\,, \tag{2.25}$$

where

$$\begin{aligned}
R^a &= d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b\omega_c\,,\\
T^a &= de^a + \epsilon^{abc}\omega_b e_c + \frac{1}{4}\bar{\psi}^i\Gamma^a\psi^i\,,
\end{aligned} \tag{2.26}$$

are the Lorentz curvature and supertorsion curvature, respectively. On the other hand,

$$\begin{aligned}
\nabla\psi^i &= d\psi^i + \frac{1}{2}\omega^a\Gamma_a\psi^i + A^{ij}\psi^j\,,\\
F^{ij} &= dA^{ij} + A^{ik}A^{kj}\,,\\
G^{ij} &= dC^{ij} + C^{ik}A^{kj} + A^{ik}C^{kj} - \bar{\psi}^i\psi^j\,.
\end{aligned} \tag{2.27}$$

Let us note that the two-form curvature related to the automorphism gauge fields has no longer spinor fields as in the super-Lorentz case.

The CS supergravity action can be written considering the non-vanishing components of the invariant tensor (2.23) and the gauge connection one-form (2.24),

$$\begin{aligned}
I^{(2+1)}_{CS} = \frac{k}{4\pi}\int \mu_0\left[\omega^a d\omega_a + \frac{1}{3}\epsilon_{abc}\omega^a\omega^b\omega^c - \frac{1}{2}\mathcal{G}(A^{ij})\right]\\
+\mu_1\left[2e^a R_a - \bar{\psi}^i\nabla\psi^i - \frac{1}{2}C^{ij}F^{ji}\right],
\end{aligned} \tag{2.28}$$

with

$$\mathcal{G}\left(A^{ij}\right) = A^{ij}dA^{ji} + \frac{2}{3}A^{ik}A^{km}A^{mi}\,. \tag{2.29}$$

Notice that the term proportional to μ_0 contains the exotic Lagrangian plus contributions coming from the automorphism gauge fields. Unlike the $\mathcal{N}$-extended AdS supergravity theories, the gravitini do not appear in the exotic sector.

By construction, the CS action (2.28) is invariant under the gauge transformation $\delta A = D\lambda = d\lambda + [A,\lambda]$. In particular, the action is invariant under the following local supersymmetry transformation laws

$$\begin{aligned}
\delta\omega^{ab} &= 0\,,\\
\delta e^a &= \frac{1}{2}\bar{\epsilon}^i\Gamma^a\psi^i\,,\\
\delta\psi^i &= \nabla\epsilon^i\,,\\
\delta A^{ij} &= 0\,,\\
\delta C^{ij} &= -2\bar{\psi}^{[i}\epsilon^{j]}\,.
\end{aligned} \tag{2.30}$$

The new procedure introduced here, allowing us to recover the $\mathcal{N}$-extended Poincaré supergravity theory, can be generalized to obtain the asymptotic symmetry of the $\mathcal{N}$-extended Poincaré supergravity. In the next section, we show first how to obtain the $\mathcal{N}=1$ super-BMS$_3$ algebra and then we naturally extend our study to the $\mathcal{N}$-extended case.

3. $\mathcal{N}$-extended super-BMS$_3$ algebra from $\mathcal{N}$-extended super-Virasoro algebra

It is well known that the BMS$_3$ symmetry emerges as a suitable contraction of the asymptotic symmetry of the AdS gravity, which is given by two copies of the Virasoro algebra. Analogously, the supersymmetric extension of the BMS$_3$ algebra comes by performing an Inönü-Wigner contraction to appropriate superconformal algebras [44,46].

In this section, we show a new way to obtain the $\mathcal{N}$-extended super-BMS$_3$ algebra from only one copy of the $\mathcal{N}$-extended super Virasoro algebra. This procedure corresponds to a supersymmetric extension of the results presented in [26]. In particular, considering the same semigroup of the previous section, we obtain the $\mathcal{N}=1,2$ and 4 super-BMS$_3$ algebra whose finite subalgebra is the $\mathcal{N}$-extended super-Poincaré one.

3.1. $\mathcal{N}=1$ super-BMS$_3$ algebra

The starting point of our construction is the super-Virasoro algebra, which we will denote as $\mathfrak{svir}$, whose (anti)-commutation relations are given by

$$\begin{aligned}
[\ell_m,\ell_n] &= (m-n)\,\ell_{m+n} + \frac{c}{12}m\left(m^2-1\right)\delta_{m+n,0}\,,\\
[\ell_m,\mathcal{Q}_r] &= \left(\frac{m}{2}-r\right)\mathcal{Q}_{m+r}\,,\\
\{\mathcal{Q}_r,\mathcal{Q}_s\} &= \ell_{r+s} + \frac{c}{6}\left(r^2-\frac{1}{4}\right)\delta_{r+s,0}\,.
\end{aligned} \tag{3.1}$$

Let us note that the super-Lorentz algebra corresponds to a finite subalgebra of the super-Virasoro one. Indeed, the three-dimensional super-Lorentz algebra is spanned by the generators $\ell_0,\ell_1,\ell_{-1},\mathcal{Q}_{\pm\frac{1}{2}}$ which are related to the super-Lorentz generators through the following change of basis:

$$\begin{aligned}
\ell_{-1} = -\sqrt{2}M_0\ ,\ \ \ell_1 = \sqrt{2}M_1\ \ ,\ \ \ell_0 = M_2\,,\\
\mathcal{Q}_{-\frac{1}{2}} = \sqrt{2}\tilde{Q}_+\ ,\ \ \mathcal{Q}_{\frac{1}{2}} = \sqrt{2}\tilde{Q}_-\,.
\end{aligned} \tag{3.2}$$

Let us consider now $S^{(2)}_E = \{\lambda_0,\lambda_1,\lambda_2,\lambda_3\}$ whose elements satisfy (2.17). After extracting a resonant subalgebra of $S^{(2)}_E\times\mathfrak{svir}$ and performing a 0_s-reduction, a new set of generators is obtained.

In fact, the expanded algebra consists of the set of generators $\{\mathcal{J}_m, \mathcal{P}_m, \mathcal{G}_r, c_1, c_2\}$ which are related to the super-Virasoro ones through the semigroup elements in the following way:

$$\begin{aligned} \mathcal{J}_m &= \lambda_0 \ell_m\,, \;\; c_1 = \lambda_0 c\,, \\ \mathcal{P}_m &= \lambda_2 \ell_m\,, \;\; c_2 = \lambda_2 c\,, \\ \mathcal{G}_r &= \lambda_1 \mathcal{Q}_r\,. \end{aligned} \tag{3.3}$$

Using the (anti-)commutators of the super-Virasoro algebra together with the multiplication law of the semigroup (2.17), one find that the non-vanishing (anti-)commutation relations of the expanded algebra are

$$\begin{aligned} [\mathcal{J}_m, \mathcal{J}_n] &= (m-n)\,\mathcal{J}_{m+n} + \frac{c_1}{12} m \left(m^2-1\right) \delta_{m+n,0}\,, \\ [\mathcal{J}_m, \mathcal{P}_n] &= (m-n)\,\mathcal{P}_{m+n} + \frac{c_2}{12} m \left(m^2-1\right) \delta_{m+n,0}\,, \\ [\mathcal{J}_m, \mathcal{G}_r] &= \left(\frac{m}{2}-r\right) \mathcal{G}_{m+r}\,, \\ \{\mathcal{G}_r, \mathcal{G}_s\} &= \mathcal{P}_{r+s} + \frac{c_2}{6}\left(r^2-\frac{1}{4}\right)\delta_{r+s,0}\,. \end{aligned} \tag{3.4}$$

The superalgebra obtained corresponds to the most generic super-BMS_3 algebra allowing two central charges [39,74]. In particular, the central charges are associated to two terms in the CS Poincaré supergravity action (2.28). Indeed, $c_1 = 12k\mu_0$ is related to the exotic CS term, while $c_2 = 12k\mu_1$ is associated to the Einstein-Hilbert term.

Note that the Poincaré superalgebra is spanned by $\mathcal{J}_0, \mathcal{J}_1, \mathcal{J}_{-1}, \mathcal{P}_0, \mathcal{P}_1, \mathcal{P}_{-1}$ and $\mathcal{G}_{\frac{1}{2}}, \mathcal{G}_{-\frac{1}{2}}$. This can be seen explicitly considering the following change of basis:

$$\begin{aligned} &\mathcal{J}_{-1} = -\sqrt{2} J_0\;\;, \;\; \mathcal{J}_1 = \sqrt{2} J_1 \;\;\;\;, \;\; \mathcal{J}_0 = J_2\,, \\ &\mathcal{P}_{-1} = -\sqrt{2} P_0\;\;, \;\; \mathcal{P}_1 = \sqrt{2} P_1 \;\;\;\;, \;\; \mathcal{P}_0 = P_2\,, \\ &\mathcal{G}_{-\frac{1}{2}} = \sqrt{2} Q_+\;\;\;, \;\; \mathcal{G}_{\frac{1}{2}} = \sqrt{2} Q_-\,. \end{aligned} \tag{3.5}$$

Then, the super-BMS_3 algebra (3.4) is the infinite-dimensional lift of the three-dimensional Poincaré superalgebra.

Interestingly, we have used the same semigroup required to obtain the super-Poincaré algebra from super-Lorentz. This show that the particular procedure used to obtain a superalgebra can also be used to derive its asymptotic symmetry. Although a similar result has been obtained at the bosonic level for Poincaré and Maxwell CS gravity [26,36], this is the first result at the supersymmetric level showing that the S-expansion method can be generalized to asymptotic symmetries.

3.2. $\mathcal{N}=2$ super-BMS_3 algebra

The extension to $\mathcal{N}=2$ super-BMS_3 algebra requires a more subtle treatment. In particular, we will focus only in the $\mathcal{N} = (2,0)$ case. Although we can extend our procedure to the $(1,1)$ super-BMS_3 algebra, this would require to consider a different starting superalgebra. In addition, the $(1,1)$ super-BMS_3 algebra has no internal symmetry generators. As we shall see, the $(2,0)$ super-BMS_3 algebra obtained here corresponds to a supersymmetric extension of the BMS_3 algebra endowed with a $\hat{\mathfrak{u}}(1)\times\hat{\mathfrak{u}}(1)$ current algebra [16,34]. This is due to the fact that the $(2,0)$ Poincaré superalgebra leading to a consistent CS supergravity action has a richer algebraic structure than the $(1,1)$ case. Indeed, the $(2,0)$ Poincaré superalgebra includes an $\mathfrak{so}(2)$ automorphism algebra [18]. Such interesting behavior is inherited to the asymptotic symmetry. Recently, the authors of [43] have shown that a democratic [42] or ultra-relativistic IW contraction of the $\mathcal{N} = (2,2)$ superconformal algebra reproduces the $\mathcal{N} = (2,0)$ super-BMS_3 algebra. Here, we show that the $(2,0)$ super-BMS_3 algebra can be alternatively obtained by expanding the $\mathcal{N}=2$ super-Virasoro algebra.

The generators of the $\mathcal{N}=2$ super-Virasoro algebra, which we shall denote as $\mathfrak{svir}_{(2)}$, satisfy the following commutators

$$\begin{aligned} [\ell_m, \ell_n] &= (m-n)\,\ell_{m+n} + \frac{c}{12} m \left(m^2-1\right) \delta_{m+n,0}\,, \\ \left[\ell_m, \mathcal{Q}^i_r\right] &= \left(\frac{m}{2}-r\right) \mathcal{Q}^i_{m+r}\,, \\ [\ell_m, \mathcal{R}_n] &= -n\mathcal{R}_{m+n}\,, \\ [\mathcal{R}_m, \mathcal{R}_n] &= \frac{c}{3} m\delta_{m+n,0}\,, \\ \left[\mathcal{Q}^i_r, \mathcal{R}_m\right] &= \epsilon^{ij}\mathcal{Q}^j_{m+r}\,, \\ \left\{\mathcal{Q}^i_r, \mathcal{Q}^j_s\right\} &= \delta^{ij}\left[\ell_{r+s} + \frac{c}{6}\left(r^2-\frac{1}{4}\right)\delta_{r+s,0}\right] - 2\epsilon^{ij}(r-s)\,\mathcal{R}_{r+s}\,, \end{aligned} \tag{3.6}$$

where the central charge $c = 12k$ is associated to the CS action (2.11). Such infinite-dimensional superalgebra differs from the $\mathcal{N}=1$ super-Virasoro one by the presence of an R-symmetry generator $\mathcal{R}_m$. Let us note that the $\mathcal{N}=2$ super-Virasoro algebra can be decomposed in subspaces as

$$\mathfrak{svir}_{(2)} = V_0 \oplus V_1\,, \tag{3.7}$$

where V_0 is the bosonic subspace spanned by the Virasoro generator ℓ_m, the central charge c and the R-symmetry generator $\mathcal{R}_m$. On the other hand, V_1 is the fermionic subspace. Such subspaces satisfy a graded Lie algebra,

$$\begin{aligned} &[V_0, V_0] \subset V_0\,, \\ &[V_0, V_1] \subset V_1\,, \\ &[V_1, V_1] \subset V_0\,. \end{aligned} \tag{3.8}$$

Let $S^{(2)}_E = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3\}$ be the relevant semigroup whose elements satisfy (2.17). The next step consists in considering a 0_s-reduced resonant subalgebra of $S^{(2)}_E \times \mathfrak{svir}_{(2)}$ following the definitions of [47]. In particular, the following subset decomposition $S^{(2)}_E = S_0 \cup S_1$, with

$$\begin{aligned} S_0 &= \{\lambda_0, \lambda_2, \lambda_3\}\,, \\ S_1 &= \{\lambda_1, \lambda_3\}\,, \end{aligned} \tag{3.9}$$

is resonant since it satisfies the same subspace structure (3.8). The expanded infinite-dimensional superalgebra is generated by the set $\left\{\mathcal{J}_m, \mathcal{P}_m, \mathcal{T}_m, \mathcal{Z}_m, \mathcal{G}^i_r, c_1, c_2\right\}$ whose generators and central charges are related to the $\mathcal{N}=2$ super-Virasoro ones through:

$$\begin{aligned} &\mathcal{J}_m = \lambda_0 \ell_m\,, \;\;\; c_1 = \lambda_0 c\,, \\ &\mathcal{P}_m = \lambda_2 \ell_m\,, \;\;\; c_2 = \lambda_2 c\,, \\ &\mathcal{T}_m = \lambda_0 \mathcal{R}_m\,, \;\; \mathcal{Z}_m = \lambda_2 \mathcal{R}_m\,, \\ &\mathcal{G}^i_r = \lambda_1 \mathcal{Q}^i_r\,. \end{aligned} \tag{3.10}$$

Using the (anti-)commutation relations of the $\mathcal{N}=2$ super-Virasoro algebra along with the multiplication law of the semigroup (2.17), one find that the (anti-)commutators of the expanded superalgebra read

$$\begin{aligned} [\mathcal{J}_m, \mathcal{J}_n] &= (m-n)\,\mathcal{J}_{m+n} + \frac{c_1}{12} m \left(m^2-1\right) \delta_{m+n,0}\,, \\ [\mathcal{J}_m, \mathcal{P}_n] &= (m-n)\,\mathcal{P}_{m+n} + \frac{c_2}{12} m \left(m^2-1\right) \delta_{m+n,0}\,, \\ [\mathcal{J}_m, \mathcal{T}_n] &= -n\mathcal{T}_{m+n}\,, \qquad [\mathcal{P}_m, \mathcal{T}_n] = -n\mathcal{Z}_{m+n}\,, \\ [\mathcal{J}_m, \mathcal{Z}_n] &= -n\mathcal{Z}_{m+n}\,, \end{aligned}$$

$$
\begin{aligned}
&[\mathcal{T}_m, \mathcal{T}_n] = \frac{c_1}{3} m\delta_{m+n,0}\,, \qquad [\mathcal{T}_m, \mathcal{Z}_n] = \frac{c_2}{3} m\delta_{m+n,0}\,, \\
&[\mathcal{J}_m, \mathcal{G}_r] = \left(\frac{m}{2} - r\right) \mathcal{G}_{m+r}\,, \\
&\left[\mathcal{Q}^i_r, \mathcal{T}_m\right] = \epsilon^{ij} \mathcal{Q}^j_{m+r}\,, \\
&\left\{\mathcal{G}^i_r, \mathcal{G}^j_s\right\} = \delta^{ij}\left[\mathcal{P}_{r+s} + \frac{c_2}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\right] + 2\epsilon^{ij}(r-s)\,\mathcal{Z}_{r+s}\,.
\end{aligned}
\tag{3.11}
$$

The infinite-dimensional superalgebra obtained corresponds to the $\mathcal{N} = (2,0)$ super-BMS$_3$ algebra. Let us note that the (anti-)commutator of the supercharges closes to a combination of $\mathcal{P}$, central charge c_2 and $\mathcal{Z}$. The superalgebra obtained here is found to be spanned by a supersymmetric extension of the enhanced asymptotic symmetry algebra of 2+1 dimensional flat space which is given by the BMS$_3$ algebra endowed with a $\hat{\mathfrak{u}}(1) \times \hat{\mathfrak{u}}(1)$ current algebra [16,34]. The explicit $\hat{\mathfrak{u}}(1)$ current generators $\mathfrak{k}_m$ and $\bar{\mathfrak{k}}_m$ appear after the redefinitions

$$
\mathcal{T}_m = \mathfrak{k}_m - \bar{\mathfrak{k}}_{-m}\,, \qquad \mathcal{Z}_m = \epsilon\left(\mathfrak{k}_m + \bar{\mathfrak{k}}_{-m}\right). \tag{3.12}
$$

In particular, (3.11) is recovered in the limit $\epsilon \to 0$.

Let us note that the (2,0) super-Poincaré algebra is spanned by $\left\{\mathcal{J}_m, \mathcal{P}_n, \mathcal{G}^i_r, \mathcal{T}_0, \mathcal{Z}_0\right\}$ with $m,n = 0,\pm 1$ and $r = \pm\frac{1}{2}$. In fact, the (anti)-commutation relations (2.21)-(2.22) for $\mathcal{N} = (2,0)$ appear explicitly after the redefinitions

$$
\begin{aligned}
&\mathcal{J}_{-1} = -\sqrt{2}J_0\;, \quad \mathcal{J}_1 = \sqrt{2}J_1\;, \quad \mathcal{J}_0 = J_2\,, \\
&\mathcal{P}_{-1} = -\sqrt{2}P_0\;, \quad \mathcal{P}_1 = \sqrt{2}P_1\;, \quad \mathcal{P}_0 = P_2\,, \\
&\mathcal{G}^i_{-\frac{1}{2}} = \sqrt{2}Q^i_+\;, \quad \mathcal{G}^i_{\frac{1}{2}} = \sqrt{2}Q^i_-\;, \\
&\mathcal{T}_0 = -T\;, \quad \mathcal{Z}_0 = -Z\,.
\end{aligned}
\tag{3.13}
$$

Then, the $\mathcal{N} = 2$ super-BMS$_3$ algebra (3.11) is the infinite-dimensional lift of the three-dimensional (2,0) Poincaré superalgebra endowed with an automorphism generator T and central charge Z. Let us note that the $\mathcal{N} = 2$ super-BMS$_3$ given by (3.11) can be alternatively derived from the $\mathcal{N} = 4$ one appearing in [46]. Indeed the (anti-)commutation relations (3.11) can be easily obtained from $\mathcal{N} = 4$ super BMS$_3$ of [46] after setting some fermionic generators to zero. An inequivalent $\mathcal{N} = 2$ super-BMS$_3$ algebra can also be obtained considering a "despotic" [42] contraction of the $\mathcal{N} = (2,2)$ superconformal algebra.

3.3. $\mathcal{N} = 4$ super-BMS$_3$ algebra

For completeness, we extend our construction to the $\mathcal{N} = 4$ case. Obtaining a $\mathcal{N} = 4$ super-BMS$_3$ algebra, following our procedure, requires to S-expand a $\mathcal{N} = 4$ super-Virasoro algebra. Here, we shall focus our attention to the derivation of a super-BMS$_3$ algebra whose finite subalgebra is not only the $\mathcal{N} = 4$ super-Poincaré algebra but also the internal algebra.

The $\mathcal{N} = 4$ super-Virasoro algebra reads as [75]:

$$
\begin{aligned}
&[\ell_m, \ell_n] = (m-n)\,\ell_{m+n} + \frac{c}{12} m\left(m^2 - 1\right)\delta_{m+n,0}\,, \\
&\left[\ell_m, \mathcal{Q}^{i,\pm}_r\right] = \left(\frac{m}{2} - r\right)\mathcal{Q}^{i,\pm}_{m+r}\,, \\
&\left[\ell_m, \mathcal{R}^a_n\right] = -n\mathcal{R}^a_{m+n}\,, \\
&\left[\mathcal{R}^a_m, \mathcal{R}^b_n\right] = i\epsilon^{abc}\mathcal{R}^c_{m+n} + \frac{c}{12} m\delta^{ab}\delta_{m+n,0}\,, \\
&\left[\mathcal{R}^a_m, \mathcal{Q}^{i,+}_r\right] = -\frac{1}{2}\left(\sigma^a\right)^i_j \mathcal{Q}^{j,+}_{m+r}\,, \\
&\left[\mathcal{R}^a_m, \mathcal{Q}^{i,-}_r\right] = \frac{1}{2}\left(\bar{\sigma}^a\right)^i_j \mathcal{Q}^{j,-}_{m+r}\,, \\
&\left\{\mathcal{Q}^{i,+}_r, \mathcal{Q}^{j,-}_s\right\} = \delta^{ij}\left[\ell_{r+s} + \frac{c}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\right] \\
&\qquad - (r-s)\left(\sigma^a\right)_{ij}\mathcal{R}^a_{r+s}\,,
\end{aligned}
\tag{3.14}
$$

where $i,j = 1,2$; $a,b,c = 1,2,3$ and $\bar{\sigma}^a_{ij} = \sigma^a_{ji}$ are the Pauli matrices. It is important to emphasize that the superalgebra (3.14) considered here contains only one set of Virasoro generators ℓ_m. In particular, the generators $\left\{\ell_m, \mathcal{Q}^{i,\pm}_r, \mathcal{R}^a_0\right\}$ with $m = 0,\pm 1$ and $r = \pm\frac{1}{2}$ satisfy a finite subalgebra which corresponds to an $\mathcal{N} = 4$ super-Lorentz.

Let $S^{(2)}_E = \{\lambda_0, \lambda_1, \lambda_2, \lambda_3\}$ be the relevant semigroup with the multiplication law given by (2.17). Considering the resonant decomposition (2.18) and applying a 0_s-reduction resonant $S^{(2)}_E$-expansion of the $\mathcal{N} = 4$ super Virasoro algebra we find a bigger infinite-dimensional superalgebra spanned by

$$
\left\{\mathcal{J}_m, \mathcal{P}_m, \mathcal{T}^a_m, \mathcal{Z}^a_m, \mathcal{G}^{i,\pm}_r, c_1, c_2\right\}. \tag{3.15}
$$

Such generators are related to the super-Virasoro ones through the semigroup elements as

$$
\begin{aligned}
&\mathcal{J}_m = \lambda_0\ell_m\,, \qquad c_1 = \lambda_0 c\,, \\
&\mathcal{P}_m = \lambda_2\ell_m\,, \qquad c_2 = \lambda_2 c\,, \\
&\mathcal{T}^a_m = \lambda_0\mathcal{R}^a_m\,, \qquad \mathcal{Z}^a_m = \lambda_2\mathcal{R}^a_m\,, \\
&\mathcal{G}^{i,\pm}_r = \lambda_1\mathcal{Q}^{i,\pm}_r\,.
\end{aligned}
\tag{3.16}
$$

Using the multiplication law of the semigroup (2.17) and the (anti-)commutation relations of the $\mathcal{N} = 4$ super-Virasoro algebra, one find that the expanded generators satisfy the following non-vanishing (anti-)commutators

$$
\begin{aligned}
&[\mathcal{J}_m, \mathcal{J}_n] = (m-n)\,\mathcal{J}_{m+n} + \frac{c_1}{12} m\left(m^2 - 1\right)\delta_{m+n,0}\,, \\
&[\mathcal{J}_m, \mathcal{P}_n] = (m-n)\,\mathcal{P}_{m+n} + \frac{c_2}{12} m\left(m^2 - 1\right)\delta_{m+n,0}\,, \\
&\left[\mathcal{J}_m, \mathcal{T}^a_n\right] = -n\mathcal{T}^a_{m+n}\,, \qquad \left[\mathcal{P}_m, \mathcal{T}^a_n\right] = -n\mathcal{Z}^a_{m+n}\,, \\
&\left[\mathcal{J}_m, \mathcal{Z}^a_n\right] = -n\mathcal{Z}^a_{m+n}\,, \\
&\left[\mathcal{T}^a_m, \mathcal{T}^b_n\right] = i\epsilon^{abc}\mathcal{T}^c_{m+n} + \frac{c_1}{12} m\delta^{ab}\delta_{m+n,0}\,, \\
&\left[\mathcal{T}^a_m, \mathcal{Z}^b_n\right] = i\epsilon^{abc}\mathcal{Z}^c_{m+n} + \frac{c_2}{12} m\delta^{ab}\delta_{m+n,0}\,, \\
&\left[\mathcal{J}_m, \mathcal{G}^{i,\pm}_r\right] = \left(\frac{m}{2} - r\right)\mathcal{G}^{i,\pm}_{m+r}\,, \\
&\left[\mathcal{T}^a_m, \mathcal{G}^{i,+}_r\right] = -\frac{1}{2}\left(\sigma^a\right)^i_j\mathcal{G}^{j,+}_{m+r}\,, \qquad \left[\mathcal{T}^a_m, \mathcal{G}^{i,-}_r\right] = \frac{1}{2}\left(\bar{\sigma}^a\right)^i_j\mathcal{G}^{j,-}_{m+r}\,, \\
&\left\{\mathcal{G}^{i,+}_r, \mathcal{G}^{j,-}_s\right\} = \delta^{ij}\left[\mathcal{P}_{r+s} + \frac{c_2}{6}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0}\right] \\
&\qquad - (r-s)\left(\sigma^a\right)_{ij}\mathcal{Z}^a_{r+s}\,.
\end{aligned}
\tag{3.17}
$$

The S-expanded algebra corresponds to a $\mathcal{N} = 4$ supersymmetric extension of the BMS$_3$ algebra. In particular, the (anti-)commutators of such infinite-dimensional symmetry close into a linear combination of $\mathcal{P}$ and their respective central charges and $\mathcal{Z}^a$ generators. Interestingly, one can show that $\mathfrak{su}(2)$ current generators $\mathfrak{k}^a_m$ and $\bar{\mathfrak{k}}^a_m$ appear after the redefinitions

$$
\mathcal{T}^a_m = \lim_{\epsilon\to 0}\left(\mathfrak{k}^a_m - \bar{\mathfrak{k}}^a_{-m}\right), \qquad \mathcal{Z}_m = \lim_{\epsilon\to 0}\epsilon\left(\mathfrak{k}^a_m + \bar{\mathfrak{k}}^a_{-m}\right). \tag{3.18}
$$

Note that the $\mathcal{N} = 4$ super-BMS$_3$ obtained here contains not only the $\mathcal{N} = 4$ super-Poincaré algebra as the finite subalgebra, but also the internal algebra generated by the $\mathcal{T}^a_m$ generators. In fact, the set $\left\{\mathcal{J}_m, \mathcal{P}_n, \mathcal{G}_r^{i,\pm}, \mathcal{T}_0^a, \mathcal{Z}_0^a\right\}$, with $m, n = 0, \pm 1$ and $r = \pm\frac{1}{2}$, generates the $\mathcal{N} = 4$ super-Poincaré algebra endowed with an internal algebra. On the other hand, the central charges $c_1 = 12k\mu_0$ and $c_2 = 12k\mu_1$ are related to the CS level (2.28).

Alternative approaches have been considered in [44,46] where alternative $\mathcal{N} = 4$ super-BMS$_3$ algebras have been presented. In particular, the supersymmetric extension of the BMS$_3$ algebra of [44,46] contains R-symmetry generators $\mathcal{R}_m$ which come from two copies of the $\mathcal{N} = 2$ super Virasoro algebra. Since for our construction we are considering only one $\mathcal{N} = 4$ super-Virasoro algebra, the final $\mathcal{N} = 4$ super-BMS$_3$ contains the presence of internal symmetry generators $\mathcal{T}_m^a$ instead of $\mathcal{R}_m$ generators. As was noticed in [18], the presence of automorphism generators are essential in order to define a non-degenerate invariant inner product which allows the formulation of a CS supergravity action.

4. Conclusions

In this paper we have presented a novel approach to obtain the $\mathcal{N}$-extended Poincaré supergravity and its respective asymptotic symmetry: the $\mathcal{N}$-extended super-BMS$_3$ algebra. This alternative approach is based on the semigroup expansion method. In particular, we have shown that the $\mathcal{N}$-extended super-Poincaré algebra with both central and automorphism generators appears by expanding the super Lorentz with a particular semigroup $S_E^{(2)}$. Interestingly, three-dimensional flat supergravity appears naturally from an exotic supersymmetric theory based only on the spin-connection. This peculiarity manifests itself only in three spacetime dimensions since there is the same number of Lorentz and boost generators allowing us to identify the expanding Lorentz fields as vielbein.

Remarkably, we have extended our results to infinite-dimensional algebras to get the $\mathcal{N}$-extended super-BMS$_3$ algebra for $\mathcal{N} = (1, 2, 4)$. It is worth it to mention that such supersymmetric extensions of the BMS$_3$ symmetry are obtained by expanding one Virasoro superalgebra using the same finite semigroup $S_E^{(2)}$ as we can see in the following diagram:

$\mathcal{N}$-extended super Lorentz	infinite-dimensional lift $\longrightarrow$	$\mathcal{N}$-extended super Virasoro
$\downarrow\ S_E^{(2)}$		$\downarrow\ S_E^{(2)}$
$\mathcal{N}$-extended super Poincaré	infinite-dimensional lift $\longrightarrow$	$\mathcal{N}$-extended super-BMS$_3$

Of particular interest are the $\mathcal{N} > 1$ super-BMS$_3$ algebras obtained here since they are not only centrally extended but also endowed with internal symmetry algebra. Interestingly, we have shown that $\mathcal{N}$-extended Poincaré superalgebras with both central and automorphism charges [18] appear as finite subalgebras of the $\mathcal{N}$-extended super-BMS$_3$ constructed here. It is interesting to note that the $\mathcal{N} = 2$ super-BMS$_3$ presented here can be easily obtained from the $\mathcal{N} = 4$ one presented in [46] after setting some fermionic generators to zero.

Our results are not only a supersymmetric generalization of those presented in [26] but could also be extended to other infinite-dimensional supersymmetries. It has been recently introduced in [26], using the S-expansion procedure, an enlarged and deformed BMS$_3$ algebra which can be seen as a infinite-dimensional lift of the Maxwell algebra. Interestingly, this new infinite-dimensional algebra results to be the corresponding asymptotic symmetry of the three-dimensional CS gravity for the Maxwell algebra [36]. Subsequently, in [38], a semi-simple enlargement of the BMS$_3$ algebra has been presented as the asymptotic symmetry of the AdS-Lorentz CS gravity. Then, motivated by these recent results, it would be interesting to explore the supersymmetric extension of these deformed and enlarged BMS$_3$ algebras using the same methodology considered here [work in progress]. One could expect that such supersymmetrization is the corresponding asymptotic symmetry of a CS supergravity [72] in three spacetime dimensions for the Maxwell and AdS-Lorentz superalgebra.

Another natural generalization of our results is the extension of our procedure to the complete family of super Maxwell like algebras [76,77]. It has been pointed out in [26] that the BMS$_3$ and deformed BMS$_3$ belong to a larger family of infinite-dimensional symmetry. One could expect to obtain the complete family of infinite-dimensional $\mathcal{N}$-extended superalgebras in which the super-BMS$_3$ is a particular case.

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