



# Resonant superalgebras and $\mathcal{N} = 1$ supergravity theories in three spacetime dimensions

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## ABSTRACT

We explore  $\mathcal{N} = 1$  supersymmetric extensions of algebras going beyond the Poincaré and AdS ones in three spacetime dimensions. Besides reproducing two known examples, we present new superalgebras, which all correspond to supersymmetric extensions with one fermionic charge  $Q_\alpha$  concerning the so-called resonant algebras being characterized by the presence of an additional bosonic generator  $Z_a$ . We point out particular requirements that superalgebras have to satisfy to be successfully incorporated within valid supergravity actions. The presented algebraic and Lagrangian framework helps us better understand the relations between the various supergravity and supersymmetric Chern-Simons actions invariant under diverse resonant superalgebras.

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## 1. Introduction

It is well known that three-dimensional supergravity theory [1] with the presence of cosmological constant can be described by the means of the Chern-Simons (CS) action and AdS superalgebra [2]. At the level of symmetry, the limit of vanishing cosmological constant can be performed through the Inönü-Wigner contraction procedure leading to the Poincaré superalgebra [3]. Subsequently, to transit beyond the three-dimensional  $\mathcal{N} = 1$  AdS and Poincaré supergravity theories towards more general models and frameworks, various extensions were studied. Among others, this concerns the incorporation of  $\mathcal{N}$ -extended charges, coupling to matter, higher-spin coupling, non-relativistic and ultra-relativistic limits (for examples see [4–17]). In this article, we will explore another interesting path that leads through the enlarged symmetries. This particular type of extension originates from the so-called Maxwell algebra, which appears to describe a Minkowski spacetime in the presence of a constant electromagnetic background [18,19]. Subsequently, the Maxwell symmetry, seen as extension and deformation of the Poincaré symmetry, has been useful to recover standard General Relativity from CS and Born-Infeld (BI) gravity theories [20,21]. In three spacetime dimensions, the additional gauge field appearing for the Maxwell algebra modifies not only the vacuum of the theory but also the asymptotic sector [22]. More recently, it was shown that the minimal massive three-dimensional gravity [23] could be seen as a particular case of a generalized minimal massive gravity theory, which appears as a spontaneous breaking of a local symmetry of the CS gravity for the Maxwell algebra [24] (also known as Hietarinta algebra [25,26]). Interestingly, the Maxwell (super)algebra can be alternatively obtained as the Inönü-Wigner contraction of the so-called Soroka-Soroka<sup>1</sup> (super)algebra [27,28]. The latter has been useful to recover diverse Lovelock gravity theories [29–31] from CS and BI theories. Other vast applications of the Maxwell and Soroka-Soroka algebras can be found in [32–43].

Both Maxwell and Soroka-Soroka algebras (and other Maxwell-like algebras [44]) can be alternatively obtained through the semigroup expansion method [45]. Indeed, they appear as the “resonant” expansions of the AdS algebra [46]. A semigroup expansion is said to be resonant when the decomposition of the semigroup satisfies the same structure as the subspaces of the original (super)algebra. The expanded algebras obtained by expanding a Lie algebra considering the resonant condition can be seen as algebraic structures that are reflecting (resonating) the original algebra and whose Lie algebra structure constants can be written directly in terms of the original algebra structure constants and the semigroup elements. The expansion procedure based on the semigroups has been of particular interest in

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<sup>1</sup> Also denoted as AdS-Lorentz algebra or semi-simple enlargement of the Poincaré algebra.

gravity context since it not only allows us to obtain novel algebras from a known one but also provides us the non-vanishing components of the invariant tensor crucial for the construction of CS and BI actions.

Recently, it was shown in [47] that the Maxwell and Soroka-Soroka algebras are not the only algebras that can be obtained as the resonant expansion of the AdS algebra. The family of expanded algebras obtained by expanding the AdS algebra along the resonant condition has been denoted as resonant algebras. Such symmetries are characterized by the presence of an additional bosonic generator  $Z_a$  which appears as a consequence of expanding the Lorentz generator of the AdS algebra. In particular, for the set of generators  $J_a, P_a, Z_a$ , the whole family consists of 6 cases, denoted as  $\mathfrak{B}_4, \tilde{\mathfrak{B}}_4, B_4, \mathfrak{C}_4, \tilde{\mathfrak{C}}_4, C_4$  (with  $\mathfrak{C}_4$  and  $\mathfrak{B}_4$  being Soroka-Soroka and Maxwell algebras, respectively). The resonant algebras turned out to be of particular interest in case of the topological insulators [48]. A detailed study about the number of possible resonant algebras depending on the generator content has then been presented in [49]. Now, in this paper, we are going to open a similar discussion in the context of the superalgebras.

The supersymmetric extension of the resonant algebras has only been studied for the case of the Maxwell and Soroka-Soroka algebras. This concerns cases with one and two fermionic charges, and with the presence of additional bosonic content [50–58]. Here, we would like to explore the  $\mathcal{N} = 1$  supersymmetric extension (with one fermionic charge) of the complete set of JPZ resonant algebras. By analyzing the super-Jacobi identities we show that some resonant algebras can present more than one supersymmetric extension scheme. We include the CS actions invariant under all of the respective resonant superalgebras. There are several motivations to do so. On one hand, higher-dimensional supergravity (as its  $D = 11$  version) requires the presence of an additional bosonic gauge field. Then, study the possible supersymmetric extensions of enlarged symmetry in three spacetime dimensions could lead to interesting toy models sharing many properties with higher-dimensional theories. On the other hand, due to the diverse applications of the resonant algebras and in particular the Maxwell algebra, it seems pertinent to explore their supersymmetric versions.

The paper is organized as follows: in Section 2, we briefly review the Poincaré and AdS CS supergravity theories in three spacetime dimensions. Section 3 and 4 contain our main results. In Section 3, we present the supersymmetric extensions of the resonant algebras. Beside known Maxwell-like examples, we present the novel superalgebras sharing a Poincaré-like bosonic sub-structure. Additionally, we discuss the requirements for the valid supersymmetric extensions of the resonant algebras. In Section 4, we construct the general super CS actions for all eight resonant superalgebras. We show that among them altogether five supersymmetric extensions are good candidates to construct supergravity actions. Section 5 concludes our work with a discussion about possible future developments.

## 2. Poincaré and AdS supergravities

In this section, we briefly review the AdS superalgebra and its vanishing cosmological constant limit: the Poincaré superalgebra. We also discuss the construction of the three-dimensional CS supergravity actions invariant under these superalgebras. This will help us to highlight crucial ingredients and details needed for the later introduction of the resonant superalgebras.

In three spacetime dimensions, the AdS superalgebra is given by  $\mathfrak{osp}(2|1) \times \mathfrak{sp}(2)$ . It is spanned by the set of generators  $\{J_a, P_a, Q_\alpha\}$ , which satisfy the following (anti-)commutation relations:

$$\begin{aligned}
 [J_a, J_b] &= \epsilon_{abc} J^c, \\
 [J_a, P_b] &= \epsilon_{abc} P^c, \\
 [P_a, P_b] &= \frac{1}{\ell^2} \epsilon_{abc} J^c, \\
 [J_a, Q_\alpha] &= \frac{1}{2} (\gamma_a)_\alpha^\beta Q_\beta, \\
 [P_a, Q_\alpha] &= \frac{1}{2\ell} (\gamma_a)_\alpha^\beta Q_\beta, \\
 \{Q_\alpha, Q_\beta\} &= -(\gamma^a C)_{\alpha\beta} P_a - \frac{1}{\ell} (\gamma^a C)_{\alpha\beta} J_a,
 \end{aligned} \tag{1}$$

where  $a, b, c = 0, 1, 2$  are the Lorentz indices raised and lowered with the Minkowski metric  $\eta_{ab}$ , and  $\epsilon_{abc}$  is the Levi-Civita tensor. Here  $\alpha = 1, 2$  are spinorial indices,  $C$  is the charge conjugation matrix, and  $\gamma^a$  are the Dirac matrices in three spacetime dimensions. For later convenience in comparing different algebras, let us present schematically the generator's structure of (1) as:

$$\begin{array}{c|cc} \dots & J & P \\ \hline J & J & P \\ P & P & J \end{array} \quad \begin{array}{c|c} \dots & Q \\ \hline J & Q \\ P & Q \end{array} \quad \begin{array}{c|c} \dots & Q \\ \hline Q & P+J \end{array} \tag{2}$$

Naturally, the vanishing cosmological constant limit  $\ell \rightarrow \infty$  leads to the Poincaré superalgebra:

$$\begin{array}{c|cc} \dots & J & P \\ \hline J & J & P \\ P & P & 0 \end{array} \quad \begin{array}{c|c} \dots & Q \\ \hline J & Q \\ P & 0 \end{array} \quad \begin{array}{c|c} \dots & Q \\ \hline Q & P \end{array} \tag{3}$$

The AdS supergravity action is straightforwardly obtained from the CS model defined in three spacetime dimensions by

$$I_{CS} = \frac{k}{4\pi} \int_{\mathcal{M}} \left( A \wedge dA + \frac{1}{3} A \wedge [A, A] \right), \tag{4}$$

where  $k = 1/4G$  is the CS level of the theory related to the gravitational constant  $G$ . The gauge field  $A$  is the gauge connection one-form and  $\langle \dots \rangle$  denotes the invariant trace.

The gauge-connection for the AdS superalgebra reads

$$A = \omega^a J_a + e^a P_a + \psi^\alpha Q_\alpha, \tag{5}$$

where  $\omega^a$  is the spin-connection,  $e^a$  denotes the vielbein and  $\psi^\alpha$  corresponds to the gravitino one-form. The corresponding full curvature two-form  $F = dA + \frac{1}{2}[A, A]$  is given by

$$F = R^a J_a + T^a P_a + \mathcal{F}^\alpha Q_\alpha, \tag{6}$$

with the super-Lorentz, super-torsion, and fermionic curvatures defined as:

$$\begin{aligned} R^a &= \mathcal{R}^a + \frac{1}{\ell^2} e^a e^b + \frac{1}{2\ell} \bar{\psi} \gamma^a \psi, \\ T^a &= D_\omega e^a + \frac{1}{2} \bar{\psi} \gamma^a \psi, \\ \mathcal{F} &= \mathcal{D}_\omega \psi + \frac{1}{2\ell} e^a \gamma_a \psi. \end{aligned} \tag{7}$$

In above, we have omitted the wedge product  $\wedge$  and defined the Lorentz covariant derivative as  $D_\omega = d + \omega$ . Note, that  $\mathcal{R}^a = d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b\omega_c$  denotes the Lorentz curvature two-form,  $D_\omega e^a = de^a + \epsilon^{abc}\omega_b e_c$  defines the torsion two-form, whereas in the fermionic curvature we have  $\mathcal{D}_\omega \psi = d\psi + \frac{1}{2}\omega^a \gamma_a \psi$ .

The AdS superalgebra (1) admits the following non-vanishing components of the invariant tensor,

$$\begin{aligned} \langle J_a J_b \rangle &= \alpha_0 \eta_{ab}, \\ \langle J_a P_b \rangle &= \frac{\alpha_1}{\ell} \eta_{ab}, \\ \langle P_a P_b \rangle &= \frac{\alpha_0}{\ell^2} \eta_{ab}, \\ \langle Q_\alpha Q_\beta \rangle &= \frac{2}{\ell} (\alpha_1 - \alpha_0) C_{\alpha\beta}, \end{aligned} \tag{8}$$

where  $\alpha_0$  and  $\alpha_1$  are arbitrary constants. In order to define a proper flat limit at the level of the invariant tensor and the CS action, let us consider the following redefinition of the coupling constants,

$$\alpha_0 \rightarrow \alpha_0, \quad \alpha_1 \rightarrow \ell \alpha_1. \tag{9}$$

With this redefinition, the components of the invariant tensor (8) lead in the vanishing cosmological constant limit  $\ell \rightarrow \infty$ , to those of the Poincaré. Through considering the superalgebra (1) with its invariant tensors (8) (along with the above redefinition) and the respective gauge connection one-form (5) in the definition of the CS action (4) with  $k = \frac{1}{4G}$ , we find the most general three-dimensional  $\mathcal{N} = 1$  AdS CS supergravity action [8] to be of the form:

$$I_{CS}^{AdS} = \frac{k}{4\pi} \int \left[ \alpha_0 \left( \omega^a d\omega_a + \frac{1}{3} \epsilon^{abc} \omega_a \omega_b \omega_c + \frac{1}{\ell^2} e_a D_\omega e^a - \frac{2}{\ell} \bar{\psi} \mathcal{F} \right) + \alpha_1 \left( 2\mathcal{R}^a e_a + \frac{1}{3\ell^2} \epsilon^{abc} e_a e_b e_c + 2\bar{\psi} \mathcal{F} \right) \right]. \tag{10}$$

One can see that the AdS CS supergravity action contains two independent terms proportional to  $\alpha_0$  and  $\alpha_1$ . In particular, the  $\alpha_0$  term contains the so-called exotic Lagrangian [59], along with the torsional term plus a contribution from the gravitino. The usual three-dimensional supergravity Lagrangian with a cosmological constant is proportional to the  $\alpha_1$  term. After considering the limit  $\ell \rightarrow \infty$ , the resulting CS action

$$I_{CS}^{Poincare} = \frac{1}{16\pi G} \int \left[ \alpha_0 \left( \omega^a d\omega_a + \frac{1}{3} \epsilon^{abc} \omega_a \omega_b \omega_c \right) + \alpha_1 \left( 2\mathcal{R}^a e_a + 2\bar{\psi} \mathcal{D}_\omega \psi \right) \right], \tag{11}$$

describes the three-dimensional  $\mathcal{N} = 1$  Poincaré CS supergravity.

### 3. Supersymmetric extensions of the resonant algebras

In this section, we shall discuss the supersymmetric extensions of the resonant bosonic algebras. Such algebras were obtained through the semigroup expansion method [45] based on the underlying structure of the AdS algebra. Leaving all the details aside, effectively they all can be seen as the enlargement of the set of generators, Lorentz and translations, including additional bosonic generators.

As it was shown in [47–49], the set of  $\{J_a, P_a, Z_a\}$  leads to six different ways we can close an algebra, which we denote as  $\mathfrak{B}_4, \mathfrak{C}_4, B_4, C_4, \tilde{B}_4, \tilde{C}_4$ . Supersymmetric extensions of the  $\mathfrak{B}_4$  and  $\mathfrak{C}_4$ , known as Maxwell and Soroka-Soroka algebras respectively, have already been explored in a wide range of spacetime dimensions for various purposes [28,50–58]. Generalizations to infinite-dimensional enhancements of the Maxwell and Soroka-Soroka superalgebras, as well as the incorporation of central charges, have also been studied in [60]. In three spacetime dimensions, supersymmetric theories based on the Maxwell and Soroka-Soroka superalgebras have been constructed in [61–63]. Although the fermionic generator  $Q_\alpha$  being present in the superalgebra leads to the supersymmetric CS model, this not necessarily assures the supergravity action. For instance, the Maxwell case provides such a non-standard supersymmetric extension [51] as we lack the translation generator  $P_a$  in the outcome of the anti-commutator of  $\{Q, Q\} \neq 0$ . To achieve  $\{Q, Q\} \sim P$ , being obligatory

for supergravity, it was necessary to include yet additional fermionic charge  $\Sigma_\alpha$ . Such a scenario was called minimal supersymmetric extension. Both types of the  $\mathcal{C}_4$  superalgebras, with single and two fermionic charges, allow us to define supergravity CS actions but only the minimal one for the vanishing cosmological constant leads to an invariant CS supergravity action under the minimal Maxwell superalgebra [62,63].

Now with more algebras coming from  $JPZ$  configurations, this paper attempts to answer the question of how many valid supersymmetric versions of resonant algebras we can find for the generator content  $JPZ + Q$ . To find all the possible realizations, we must first ask which features are necessary to get a valid superalgebra, and what is required to successfully build a gauge theory of supergravity.

The minimal requirement for the supersymmetric extension of the resonant algebra should be possessing a Lorentz subalgebra spanned by  $J$  and invariance under Lorentz rotations  $J$  for all bosonic and fermionic generators. Additionally, we must include the so-called resonant condition, which organizes the outcomes of the commutation relations for the  $L=Lorentz-like$  and  $T=translational-like$  generators to assure a structure incorporated within the original AdS:

$$[L, L] \sim L, \quad [L, T] \sim T, \quad [T, T] \sim L.$$

This extends also for the fermions:  $[L, Q] \sim Q$  and  $[T, Q] \sim Q$ . Although it is not visible on the first sight, a new generator  $Z$  presented in this paper is of the Lorentzian-like type as was shown in [47–49].

Ultimately, we must require that all the generators satisfy the super-Jacobi identities in order to have true superalgebras. Naturally, we shall avoid candidates possessing relation  $\{Q, Q\} = 0$  passing Jacobi identities since they would imply the absence of fermionic contributions to the bosonic curvatures, and in turn, to the action. Note that the generator  $P_a$ , appearing in the AdS and Poincaré superalgebras, is expressed as bilinear expressions of fermionic generators  $Q$ . Such a feature assures the proper coupling of the graviton (Einstein-Hilbert) with the gravitino (Rarita-Schwinger) describing a supergravity theory. We are going to allow a much wider scope of superalgebras allowing the fermionic contributions in other sectors of the CS action different to the Einstein-Hilbert one. In that non-standard case, we would describe such action not as supergravity but as the supersymmetric Chern-Simons theory.

Extensive searches of all possibilities, aided with the *Cadabra* symbolic computer algebra system [64] have resulted ultimately in eight superalgebras, among them five leads to the supergravity formulation and three to the non-standard cases. Two represents already known non-standard supersymmetric extension of  $\mathcal{C}_4$  and  $\mathcal{B}_4$  algebras, Soroka-Soroka [28] and Maxwell [51], respectively:

$$\begin{array}{l} \mathcal{C}_4 : \quad \begin{array}{c|ccc} \dots & J & P & Z \\ \hline J & J & P & Z \\ P & P & Z & P \\ Z & Z & P & Z \end{array} \quad \begin{array}{c|c} \dots & Q \\ \hline J & Q \\ P & Q \\ Z & Q \end{array} \quad \begin{array}{c|c} \{\dots\} & Q \\ \hline Q & P+Z \end{array} \\ \\ \mathcal{B}_4 : \quad \begin{array}{c|ccc} \dots & J & P & Z \\ \hline J & J & P & Z \\ P & P & Z & 0 \\ Z & Z & 0 & 0 \end{array} \quad \begin{array}{c|c} \dots & Q \\ \hline J & Q \\ P & 0 \\ Z & 0 \end{array} \quad \begin{array}{c|c} \{\dots\} & Q \\ \hline Q & Z \end{array} \end{array}$$

On the other hand, all novel supersymmetric extensions of the resonant algebras correspond to  $C_4, \tilde{C}_4, \tilde{B}_4$  cases, which share the Poincaré-like sub-structure [47–49]:

$$\begin{array}{l} C_4 : \quad \begin{array}{c|ccc} \dots & J & P & Z \\ \hline J & J & P & Z \\ P & P & 0 & P \\ Z & Z & P & Z \end{array} \quad \begin{array}{c|c} \dots & Q \\ \hline J & Q \\ P & 0 \\ Z & Q \end{array} \quad \begin{array}{c|c} \{\dots\} & Q \\ \hline Q & P \end{array} \\ \\ \tilde{C}_4 : \quad \begin{array}{c|ccc} \dots & J & P & Z \\ \hline J & J & P & Z \\ P & P & 0 & 0 \\ Z & Z & 0 & Z \end{array} \quad \begin{array}{c|c} \dots & Q \\ \hline J & Q \\ P & 0 \\ Z & 0 \end{array} \quad \begin{array}{c|c} \{\dots\} & Q \\ \hline Q & P \end{array} \quad \textcircled{a} \\ \\ \begin{array}{c|c} \dots & Q \\ \hline J & Q \\ P & 0 \\ Z & Q \end{array} \quad \begin{array}{c|c} \{\dots\} & Q \\ \hline Q & Z \end{array} \quad \textcircled{b} \end{array}$$

$$\begin{aligned}
 \tilde{B}_4 : \quad & \begin{array}{c|ccc} \dots & J & P & Z \\ \hline J & J & P & Z \\ P & P & 0 & 0 \\ Z & Z & 0 & 0 \end{array} & \begin{array}{c|c} \dots & Q \\ \hline J & Q \\ P & 0 \\ Z & 0 \end{array} & \begin{array}{c|c} \dots & Q \\ \hline Q & P \end{array} & \textcircled{a} \\
 & & & \begin{array}{c|c} \dots & Q \\ \hline Q & Z \end{array} & \textcircled{b} \\
 & & & \begin{array}{c|c} \dots & Q \\ \hline Q & P+Z \end{array} & \textcircled{c}
 \end{aligned}$$

It should be pointed out that one can not construct a consistent supersymmetric extension with one spinor charge for the  $B_4$  algebra:

$$B_4 : \quad \begin{array}{c|ccc} \dots & J & P & Z \\ \hline J & J & P & Z \\ P & P & 0 & P \\ Z & Z & P & 0 \end{array} \quad \begin{array}{c|c} \dots & Q \\ \hline J & - \\ P & - \\ Z & - \end{array} \quad \begin{array}{c|c} \dots & Q \\ \hline Q & - \end{array}$$

As the Soroka-Soroka  $\mathcal{C}_4$  superalgebra represents the most general form without any zero entries, it is good to complete this section with its explicit form in three spacetime dimensions:

$$\begin{aligned}
 [J_a, J_b] &= \epsilon_{abc} J^c, & [J_a, P_b] &= \epsilon_{abc} P^c, \\
 [P_a, P_b] &= \epsilon_{abc} Z^c, & [J_a, Z_b] &= \epsilon_{abc} Z^c, \\
 [Z_a, Z_b] &= \frac{1}{\ell^2} \epsilon_{abc} Z^c, & [P_a, Z_b] &= \frac{1}{\ell^2} \epsilon_{abc} P^c, \\
 [J_a, Q_\alpha] &= \frac{1}{2} (\gamma_a)_\alpha^\beta Q_\beta, \\
 [P_a, Q_\alpha] &= \frac{1}{2\ell} (\gamma_a)_\alpha^\beta Q_\beta, \\
 [Z_a, Q_\alpha] &= \frac{1}{2\ell^2} (\gamma_a)_\alpha^\beta Q_\beta, \\
 \{Q_\alpha, Q_\beta\} &= -(\gamma^a C)_{\alpha\beta} \left( \frac{1}{\ell} P_a + Z_a \right).
 \end{aligned} \tag{12}$$

This allows us to easily reproduce the explicit form for any other scheme presented in the  $JPZ + Q$  tables given earlier. All other superalgebras will represent merely alterations in existence of the particular outcomes.

It is important to clarify that we referred to the presented superalgebras as “resonant superalgebras” since they are supersymmetric extensions of the resonant algebras. However, unlike their bosonic counterparts, they have not been obtained considering the semigroup expansion method, but through the extensive brute-force search at the level of the (super)generators. Interestingly, the  $\mathcal{C}_4$  and  $\tilde{B}_4^c$  superalgebras could be seen directly as the resonant expanded superalgebras by considering particular semigroups. Indeed, they can be alternatively obtained as resonant expansions of the  $osp(2, 1) \otimes sp(2)$  superalgebra, which corresponds to the supersymmetric extension of the AdS algebra in three spacetime dimensions. Unlike the aforesaid superalgebras, the other cases cannot be obtained by resonant expansions of the AdS superalgebra. This can be seen directly from the (anti-)commutation relations of the resonant superalgebras. Indeed the AdS superalgebra is characterized by the anticommutator  $\{Q, Q\} \sim J + P$ . Then, any expanded superalgebra satisfying the AdS resonant condition should possess a similar structure like the  $\mathcal{C}_4$  and  $\tilde{B}_4^c$  superalgebras. The resonant property (in the expansion formalism) formally appears on their bosonic subalgebras, which correspond to the respective resonant algebras. The exploration of a family of superalgebras being truly resonant expansions of a known superalgebra remains an interesting open issue requiring consideration of a different original superalgebra or generalization of the present notion of superalgebra semigroup expansion.

#### 4. Supergravity and supersymmetric actions based on resonant superalgebras

In this section, we construct the supersymmetric CS actions based on the resonant superalgebras in three spacetime dimensions. Analogously to [48], we start from the explicit  $\mathcal{C}_4$  superalgebra and the corresponding CS action construction, as it provides the richest Lagrangian content. For other superalgebras, we luckily avoid reshuffling terms into various sub-invariant sectors, so it is all reducing down to either presence of the particular term in the action or its absence.

Let us consider first the gauge connection one-form

$$A = \omega^a J_a + e^a P_a + h^a Z_a + \psi^\alpha Q_\alpha, \tag{13}$$

where  $\omega^a$  is the spin-connection one-form,  $e^a$  is the vielbein,  $h^a$  is the additional gauge field related to  $Z_a$  and  $\psi$  is the gravitino.

The corresponding super-curvature two-form  $F = dA + \frac{1}{2} [A, A]$  reads

$$F = R^a J_a + T^a P_a + H^a Z_a + \mathcal{F}^\alpha Q_\alpha, \tag{14}$$

where

$$\begin{aligned}
R^a &= \mathcal{R}^a, \\
T^a &= D_\omega e^a + \frac{1}{\ell^2} \epsilon^{abc} h_b e_c + \frac{1}{2\ell} \bar{\psi} \gamma^a \psi, \\
H^a &= D_\omega h^a + \frac{1}{2\ell^2} \epsilon^{abc} h_b h_c + \frac{1}{2} \epsilon^{abc} e_b e_c + \frac{1}{2\ell} \bar{\psi} \gamma^a \psi, \\
\mathcal{F} &= D_\omega \psi + \frac{1}{2\ell} e^a \gamma_a \psi + \frac{1}{2\ell^2} h^a \gamma_a \psi.
\end{aligned}$$

The Soroka-Soroka  $\mathfrak{C}_4$  superalgebra admits the following non-vanishing components of the non-degenerate invariant tensor [61],

$$\begin{aligned}
\langle J_a J_b \rangle &= \alpha_0 \eta_{ab}, & \langle P_a P_b \rangle &= \frac{\alpha_2}{\ell^2} \eta_{ab}, \\
\langle J_a P_b \rangle &= \frac{\alpha_1}{\ell} \eta_{ab}, & \langle J_a Z_b \rangle &= \frac{\alpha_2}{\ell^2} \eta_{ab}, \\
\langle Z_a P_b \rangle &= \frac{\alpha_1}{\ell^3} \eta_{ab}, & \langle Z_a Z_b \rangle &= \frac{\alpha_2}{\ell^4} \eta_{ab}, \\
\langle Q_\alpha Q_\beta \rangle &= \frac{2}{\ell^2} (\alpha_1 - \alpha_2) C_{\alpha\beta},
\end{aligned} \tag{15}$$

where  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$  are arbitrary constants. Note that their appearance for different superalgebras is also directly associated with the form of particular  $JPZ + Q$  tables [47–49]. The particular set of the  $\alpha$ 's constants can be seen as the result of the semigroup expansion procedure applied to the original (super)algebra. In fact, one of the advantages of working with the semigroup expansion method is that it provides automatically the invariant tensors of the expanded (super)algebra. For instance, in the case of the Soroka-Soroka superalgebra, the arbitrary constants appearing in (15) are coming from the original AdS supertrace [34]. Obviously, it is not always possible to relate two (super)algebras through an expansion. One could find the non-vanishing components of the invariant tensor for a given (super)algebra, by just considering the realization of the identity  $\langle [X_A, X_B] X_C \rangle = \langle X_A [X_B, X_C] \rangle$ , where  $X_A$  are the generators of the (super)algebra.

Analogously to the AdS case, we consider the following redefinition of the arbitrary constants,

$$\alpha_0 \rightarrow \alpha_0, \quad \alpha_1 \rightarrow \ell \alpha_1, \quad \alpha_2 \rightarrow \ell^2 \alpha_2, \tag{16}$$

to have a right flat limit at the level of invariant tensors and CS action. Indeed, the non-vanishing components of the invariant tensor for the Maxwell algebra are directly obtained from (15) after the limit  $\ell \rightarrow \infty$ . Then, considering the non-vanishing components of the invariant tensor for the  $\mathfrak{C}_4$  superalgebra (15) (along with the redefinition (16)) and the respective gauge connection one-form (13) in the general expression of the CS action (4), we find the most general three-dimensional  $\mathcal{N} = 1$  CS supergravity action (up to the boundary terms) being invariant under the Soroka-Soroka  $\mathfrak{C}_4$  superalgebra:

$$\begin{aligned}
I_{CS}^{\mathfrak{C}_4} &= \frac{k}{4\pi} \int \left[ \alpha_0 \left( \omega^a d\omega_a + \frac{1}{3} \epsilon^{abc} \omega_a \omega_b \omega_c \right) + \alpha_1 \left( 2\mathcal{R}^a e_a + \frac{1}{3\ell^2} \epsilon^{abc} e_a e_b e_c + \frac{2}{\ell^2} e_a D_\omega h^a + \frac{1}{\ell^4} \epsilon^{abc} e_a h_b h_c + \frac{2}{\ell} \bar{\psi} \mathcal{F} \right) \right. \\
&\quad \left. + \alpha_2 \left( e_a D_\omega e^a + 2h_a \mathcal{R}^a + \frac{1}{\ell^2} \epsilon^{abc} e_a e_b h_c + \frac{1}{\ell^2} h_a D_\omega h^a + \frac{1}{3\ell^4} \epsilon^{abc} h_a h_b h_c - 2\bar{\psi} \mathcal{F} \right) \right]. \tag{17}
\end{aligned}$$

One can see that the CS supergravity action contains three independent sectors proportional to  $\alpha_0$ ,  $\alpha_1$  and  $\alpha_2$ . In particular, unlike the AdS case, the  $\alpha_0$  term does not contain a fermionic term and reduces to the exotic Lorentz Lagrangian  $\mathcal{L}^{exotic} = CS(\omega)$  [59]. The term proportional to  $\alpha_1$  contains the Einstein-Hilbert term, the cosmological constant term, a generalized Rarita-Schwinger term with the presence of the additional gauge field  $h^a$  in curvature  $\mathcal{F}$  and the coupling of the extra gauge field with the vielbein and spin-connection. A new contribution is proportional to  $\alpha_2$  and contains a torsional term, a generalized cosmological constant term, and a generalized Rarita-Schwinger term.

It is interesting to note that a vanishing cosmological constant limit leads to the following non-standard supersymmetric CS action,

$$I_{CS}^{\mathfrak{B}_4} = \frac{k}{4\pi} \int \left[ \alpha_0 \left( \omega^a d\omega_a + \frac{1}{3} \epsilon^{abc} \omega_a \omega_b \omega_c \right) + \alpha_1 (2\mathcal{R}^a e_a) + \alpha_2 (e_a D_\omega e^a + 2h_a \mathcal{R}^a - 2\bar{\psi} D_\omega \psi) \right]. \tag{18}$$

Although such CS action contains the fermionic gauge field, one can see that the term proportional to  $\alpha_1$  reduces only to the Einstein-Hilbert term and does not describe a proper supergravity Lagrangian. This supersymmetric CS action is invariant under the non-standard supersymmetric extension of the Maxwell algebra  $\mathfrak{B}_4$ .

Since the CS supergravity action for the  $\mathfrak{C}_4$  superalgebra (17) contains all possible terms, the other supersymmetric CS actions will be uniquely characterized by the absence of particular terms. The following table summarizes all the diverse supersymmetric CS actions constructed from the  $JPZ + Q$  resonant superalgebras:

	$\tilde{B}_4^a$	$\tilde{B}_4^b$	$\tilde{B}_4^c$	$\tilde{C}_4^a$	$\tilde{C}_4^b$	$B_4$	$C_4$	$\mathfrak{B}_4$	$\mathfrak{C}_4$
$2e_a \mathcal{R}^a$	$\alpha_1$	$\alpha_1$	$\alpha_1$	$\alpha_1$	$\alpha_1$	$\alpha_1$	$\alpha_1$	$\alpha_1$	$\alpha_1$
$\frac{2}{\ell^2} e_a D_\omega h^a$	$\alpha_1$	$\alpha_1$	$\alpha_1$				$\alpha_1$		$\alpha_1$
$\frac{1}{\ell^4} \epsilon^{abc} e_a h_b h_c$							$\alpha_1$		$\alpha_1$
$\frac{1}{3\ell^2} \epsilon^{abc} e_a e_b e_c$									$\alpha_1$
$CS(\omega)$	$\alpha_0$	$\alpha_0$	$\alpha_0$	$\alpha_0$	$\alpha_0$	$\alpha_0$	$\alpha_0$	$\alpha_0$	$\alpha_0$
$2h_a \mathcal{R}^a$	$\alpha_2$	$\alpha_2$	$\alpha_2$	$\alpha_2$	$\alpha_2$	$\alpha_2$	$\alpha_2$	$\alpha_2$	$\alpha_2$
$\frac{1}{\ell^2} h_a D_\omega h^a$				$\alpha_2$	$\alpha_2$		$\alpha_2$		$\alpha_2$
$\frac{1}{3\ell^4} \epsilon^{abc} h_a h_b h_c$				$\alpha_2$	$\alpha_2$		$\alpha_2$		$\alpha_2$
$\frac{1}{\ell^2} \epsilon^{abc} e_a e_b h_c$									$\alpha_2$
$e_a D_\omega e^a$								$\alpha_2$	$\alpha_2$
$2\bar{\psi} \mathcal{D}_\omega \psi$	$\frac{\alpha_1}{\ell}$	$-\alpha_2$	$\frac{\alpha_1}{\ell} - \alpha_2$	$\frac{\alpha_1}{\ell}$	$-\alpha_2$		$\frac{\alpha_1}{\ell}$	$-\alpha_2$	$\frac{\alpha_1}{\ell} - \alpha_2$
$\frac{1}{\ell} \bar{\psi} e^a \gamma_a \psi$									$\frac{\alpha_1}{\ell} - \alpha_2$
$\frac{1}{\ell^2} \bar{\psi} h^a \gamma_a \psi$					$-\alpha_2$		$\frac{\alpha_1}{\ell}$		$\frac{\alpha_1}{\ell} - \alpha_2$

Let us note that five superalgebras produce the Rarita-Schwinger term in the same sector as the Einstein-Hilbert term, which describes consistent supergravity. Two superalgebras include the additional contribution from the gauge field  $h^a$  to the Rarita-Schwinger and Einstein-Hilbert terms. In turn, the  $\tilde{C}_4^a$  superalgebra assures the pure supergravity action without cosmological constant ( $\alpha_1$  sector), while the extra field  $h^a$  appears only in the  $\alpha_2$  sector. Interestingly, only the Soroka-Soroka superalgebra offers the possibility to construct a supergravity action with the cosmological constant. On the other hand, although there are well-defined supersymmetric extensions of the  $\tilde{B}_4^b$ ,  $\tilde{C}_4^b$  and  $\mathfrak{B}_4$  algebras, such superalgebras reproduce non-standard supersymmetric CS actions with no fermionic contributions in the  $\alpha_1$  term. In the case of the  $B_4$  algebra there is no consistent supersymmetric extension with one fermionic charge. This gives us a purely bosonic CS action, which we left in the table for comparison.

The physical implications of the new gauge field  $h^a$  in three-dimensional supergravity theories remain an interesting open issue. At the bosonic level, the extra gauge field  $h^a$  has diverse interpretations in three spacetime dimensions depending on the choice of particular resonant algebra. Indeed, in the case of the Maxwell algebra  $\mathfrak{B}_4$  field  $h^a$  is called the Maxwell gravitational gauge field, whose generator  $Z_a$  influences both the vacuum energy and angular momentum of the stationary configuration [22]. Furthermore, such additional generator modifies also the asymptotic structure by extending and deforming the  $\mathfrak{bms}_3$  algebra. In the  $\mathfrak{C}_4$  case, the asymptotic symmetry modified by the generator  $Z_a$  is now described by three copies of the Virasoro algebra and the CS theory admits a BTZ type solution [38]. In the  $\mathfrak{B}_4$  case we can observe the interchanging role of the vielbein  $e^a$  and the extra gauge field  $h^a$ , which offers us a completely different CS theory from the physical point of view. In such interpretation, the algebra reproduces the simplest Hietarinta algebra [25], which leads to a three-dimensional CS gravity action allowing to construct actions with the spontaneous breaking of this symmetry without breaking the Poincaré invariance [24,26]. Another interesting development appears in [48], where the standard electromagnetic gauge field  $A_\mu$  gets immersed in one of the components of  $h_\mu^a$ , while setting others to zero. This establishes non-trivial description of 3D materials called topological insulators and, depending on the particular algebras, some possible modifications to the quantum Hall effect.

In three-dimensional supergravity context one requires to carry further analysis and explore the role of the additional gauge field  $h^a$ . In four spacetime dimensions, a minimal supersymmetric extension of the  $\mathfrak{B}_4$  algebra appears as a direct consequence of describing a generalized superspace in presence of a constant Abelian supersymmetric gauge field background [50]. One could start studying the boundary conditions along the lines of [22,38] for the resonant gauge field (13) to obtain the respective asymptotic symmetry of the CS supergravity theory. Furthermore, such construction could bring valuable information about bosonic solutions.

As an ending remark, one could naturally extend our results to four spacetime dimensions. This could be achieved by using the MacDowell-Mansouri formalism [65]. In particular, the geometrical formulation of supergravity in such formalism has already been presented for the Maxwell and Soroka-Soroka superalgebra in [52,55,66]. Then, one could generalize the procedure to new resonant superalgebras containing Poincaré-like subalgebras. Especially, it would be interesting to analyze the requirements for the additional gauge field  $h_a$  in order to construct dynamical actions.

#### 4.1. Flat limit and Inönü-Wigner contraction

As we have shown previously, the supersymmetric action based on the non-standard Maxwell superalgebra  $\mathfrak{B}_4$  can be obtained as a vanishing cosmological constant limit  $\ell \rightarrow \infty$  of the  $\mathfrak{C}_4$  supergravity theory. Such flat limit can be applied explicitly at the level of the (anti-)commutation relations (12), the invariant tensor (15), the curvature two-forms (14) and CS action (17). It is intriguing to see that the vanishing of the cosmological constant leads us to the non-standard supersymmetric theory whose action describes no more a supergravity theory. As we have previously discussed, such peculiarity can be avoided by considering an additional fermionic generator leading to the so-called minimal scheme. Configurations with more than one spinor charge will be the subject of some future work.

As we shall see, the flat limit is not the only limit allowing us to relate two resonant superalgebras. Indeed, various rescalings of the generators and the arbitrary constant appearing in the invariant tensor allow establishing a well-defined IW contraction at the level of the superalgebra and CS action. To this end, we consider a rescaling parameter  $\sigma$  and apply the limit  $\sigma \rightarrow \infty$ . The table below contains the different rescalings of the generators and coupling constants appearing in the  $\{\mathfrak{C}_4, \tilde{C}_4^a, \tilde{C}_4^b\}$  family which in the limit  $\sigma \rightarrow \infty$  reproduce the  $\{\mathfrak{B}_4, \tilde{B}_4^a, \tilde{B}_4^b, \tilde{B}_4^c\}$  superalgebras with their respective invariant tensors.

	$\mathfrak{C}_4 \rightarrow \mathfrak{B}_4$	$\mathfrak{C}_4 \rightarrow \tilde{\mathfrak{B}}_4^c$	$\tilde{\mathfrak{C}}_4^a \rightarrow \tilde{\mathfrak{B}}_4^a$	$\tilde{\mathfrak{C}}_4^b \rightarrow \tilde{\mathfrak{B}}_4^b$
$J_a$	$J_a \rightarrow J_a$	$J_a \rightarrow J_a$	$J_a \rightarrow J_a$	$J_a \rightarrow J_a$
$P_a$	$P_a \rightarrow \sigma P_a$	$P_a \rightarrow \sigma^2 P_a$	$P_a \rightarrow P_a$	$P_a \rightarrow P_a$
$Z_a$	$Z_a \rightarrow \sigma^2 Z_a$	$Z_a \rightarrow \sigma^2 Z_a$	$Z_a \rightarrow \sigma Z_a$	$Z_a \rightarrow \sigma^2 Z_a$
$Q_\alpha$	$Q_\alpha \rightarrow \sigma Q_\alpha$	$Q_\alpha \rightarrow \sigma Q_\alpha$	$Q_\alpha \rightarrow Q_\alpha$	$Q_\alpha \rightarrow \sigma Q_\alpha$
$\alpha_0$	$\alpha_0 \rightarrow \alpha_0$	$\alpha_0 \rightarrow \alpha_0$	$\alpha_0 \rightarrow \alpha_0$	$\alpha_0 \rightarrow \alpha_0$
$\alpha_1$	$\alpha_1 \rightarrow \sigma \alpha_1$	$\alpha_1 \rightarrow \sigma^2 \alpha_1$	$\alpha_1 \rightarrow \alpha_1$	$\alpha_1 \rightarrow \alpha_1$
$\alpha_2$	$\alpha_2 \rightarrow \sigma^2 \alpha_2$	$\alpha_2 \rightarrow \sigma^2 \alpha_2$	$\alpha_2 \rightarrow \sigma \alpha_2$	$\alpha_2 \rightarrow \sigma^2 \alpha_2$

One can see that the vanishing cosmological constant limit, relating  $\mathfrak{C}_4$  and  $\mathfrak{B}_4$  superalgebras, can be interpreted as an IW contraction process analogously to what happens in the AdS and Poincaré superalgebras. Although the other resonant superalgebras related by an IW contraction do not need to rescale the constants of the invariant tensor, one can see that the proper contraction of the CS action requires appropriate rescaling of the  $\alpha$ 's. It is interesting to point out that the presence of an additional bosonic generator concerning the AdS and Poincaré symmetries not only offers diverse ways to close a superalgebra but also leads us to several relations between the superalgebras through the deformation/contraction procedure.

## 5. Conclusions

In this paper, we have analyzed all possible  $\mathcal{N} = 1$  supersymmetric extensions of the so-called resonant algebras [47–49] by studying the super-Jacobi identities. To this purpose, we provided a list of minimal requirements for the superalgebras. We obtained not only the already known Maxwell and Soroka-Soroka superalgebras [28,51] but also novel superalgebras, which possess a Poincaré-like bosonic structure. Interestingly, some particular resonant algebras offer more than one supersymmetric extension. Altogether, for the  $JPZ + Q$  configuration, we have discovered eight different cases. However, as we have discussed, only five supersymmetric extensions of the resonant algebras reproduce consistent CS supergravity actions, whereas the other three represent non-standard examples.

Gathering all these superalgebras in one place with their respective CS actions, immediately highlights many relations, differences, and similarities between them. It is interesting to see how subtle differences in commutation relations can lead to important differences in the CS action. In particular, regarding only the Einstein-Hilbert term within  $\alpha_1$  sector, the five “good” resonant superalgebras allow us to describe diverse supergravity CS actions with and without cosmological constant and the contribution from the extra gauge field  $h_a$ . It would be worthwhile to explore the full physical implications of such an extra field. Although there are some interpretations of an additional  $h^a$  gauge field in the three-dimensional Maxwell CS gravity theory [22,24,26,48], the fermionic framework remains still an opened issue.

Extending our results to more fermionic charges seems a natural next step. Due to the presence of a second spinorial charge, one can expect a greater number of possibilities. Let us note that the presence of a second spinorial charge is not new and has been previously introduced in superstring context [67] and  $D = 11$  supergravity theory [68]. Such (minimal) supersymmetric extensions of the  $\mathfrak{B}_4$  and  $\mathfrak{C}_4$ , without considering additional bosonic content, have been recently obtained as the resonant expansions of the Lorentz superalgebra [62, 63]. As it was discussed in [62,63], unlike the non-standard case, a minimal supersymmetric extension of the Maxwell algebra  $\mathfrak{B}_4$  allows defining a consistent CS supergravity action in three spacetime dimensions. Then, it would be also interesting to explore supersymmetric extensions of other resonant algebras with more fermionic charges and bosonic content.

Finally, our results might provide valuable information about the underlying symmetry behind supergravity in higher dimensions. It would be interesting to explore the possibility to recover Cremmer-Julia-Scherk (CJS) supergravity [69] from a more general CS supergravity based on enlarged superalgebras.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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